UNIVERSIDADE FEDERAL DA BAHIA
INSTITUTO DE MATEMÁTICA
DEPARTAMENTO DE MATEMÁTICA
MAT B33 – LIMITES E DERIVADAS

LISTA DE EXERCÍCIOS(Questões de Provas I^a UNIDADE)

Limite

1. (2000 – 1) Considere a função de domínio *IR*, definida por $f(x) = \begin{cases} x^2, & x < -1 \\ -x+1, & -1 \le x < 1 \\ \log x, & x \ge 1 \end{cases}$.

1.1) Esboce o gráfico de f .

1.2) Determine se existirem: $\lim\limits_{x \to -\infty} f(x)$, $\lim\limits_{x \to +\infty} f(x)$, $\lim\limits_{x \to -1} f(x)$, $\lim\limits_{x \to 0} f(x)$ e $\lim\limits_{x \to 1} f(x)$.

2. (2000 – 1) Considerando $f(x) = \dfrac{x^2 + |x|}{x}$, analise qual o domínio em que $f(x)$ define uma função, esboce o gráfico desta função no seu domínio de definição, e determine, se existirem, os limites $\lim\limits_{x \to 0^-} f(x)$, $\lim\limits_{x \to 0^+} f(x)$ e $\lim\limits_{x \to 0} f(x)$.

3. (2005-2) Dada a função $f(x) = \dfrac{ax^2 + bx}{(x-c)^2}$,para x $\neq$ c. Calcule as constantes a, b e c de modo que: $\lim\limits_{x \to 1} f(x) = -\infty$, $\lim\limits_{x \to +\infty} f(x) = 3$ e f(4/3) = 0.

4. Calcule, se existirem, os seguintes limites:

4.01) (2000 – 1) $\lim\limits_{x \to \frac{\pi}{2}} \left(\dfrac{1}{2}\right)^{tgx}$

4.02) (1999 – 1) $\lim\limits_{x \to -3} \dfrac{x+3}{|x-3|-6}$

4.03) (2000 – 1) $\lim\limits_{x \to 1} \dfrac{x^2 - 7x + 6}{x^3 - 3x + 2}$

4.04) (1999 – 2) $\lim\limits_{x \to 1^+} \left[\dfrac{1}{1-x} - \dfrac{1}{1-x^3}\right]$

4.05) (1999 – 2) $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x+x^2} - 1}{x}$

4.06) (2000 – 1) $\lim\limits_{x \to 0} \dfrac{\sqrt{9+x} - 3}{x}$

4.07) (2000 – 1) $\lim\limits_{x \to 1} \dfrac{\sqrt{x^2+1} - \sqrt{2}}{3x^2 - 3x}$

4.08) (1999 – 2) $\lim\limits_{x \to +\infty} (x - \sqrt{x^2+1})$

4.09) (1999 – 2) $\lim\limits_{x \to -\infty} \left[\ln(\sqrt{x^2+2} + x)\right]$

4.10) (1999 – 2) $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2-3}}{\sqrt[3]{x^3+1}}$

4.11) (2000 – 1) $\lim\limits_{x \to +\infty} \dfrac{\sqrt[3]{9-x^6}}{x^3+1}$

4.12) (1999 – 1) $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2-3}}{\sqrt[3]{8x^3+1}}$

4.13) (1999 – 2) $\lim\limits_{x \to -5^+} \dfrac{5-x}{x^2+x-20}$

4.14) (1999 – 2) $\lim\limits_{x\to -1} \dfrac{2^x - 1}{x^3 - x}$

4.15) (2000 – 1) $\lim\limits_{x\to -1} \dfrac{\sqrt{3x^2 + x - 2}}{x^3 + 1}$

4.16) (2000 – 1) $\lim\limits_{x\to 0} \dfrac{1 - \cos x}{x^2}$

4.17) (1999 – 1) $\lim\limits_{x\to 0} \dfrac{tg^2 x + \operatorname{sen}^2 x}{x \operatorname{sen} x}$

4.18) (2000 – 1) $\lim\limits_{x\to 0} \dfrac{tgx - \operatorname{sen} x}{x^3}$

4.19) (1999 – 1) $\lim\limits_{x\to \pi} \dfrac{\operatorname{sen} x}{x - \pi}$

4.20) (2000 – 1) $\lim\limits_{x\to +\infty} \dfrac{\cos x}{x}$

4.21) (2000 – 1) $\lim\limits_{x\to 0^+} \left[\dfrac{1}{\ln x} \operatorname{sen} \dfrac{1}{x} \right]$

4.22) (1999 – 2) $\lim\limits_{x\to +\infty} \left(\pi^{\frac{2-x^2}{1+x}} \cos(\ln x) \right)$

4.23) (1999 – 2) $\lim\limits_{x\to +\infty} \dfrac{\operatorname{sen}(x+2)}{\sqrt[3]{x^2+2} + \sqrt[3]{x^2}}$

4.24) (2005-2) $\lim\limits_{x\to +\infty} 2^{x(\sqrt{x^2+4} - x)}$

4.25) (2005-2) $\lim\limits_{x\to 1} \left(\dfrac{2(x-1)}{sen(\pi x)} \right)^2$

4.26)(2006-1) $\lim\limits_{x\to \pi} \left(\dfrac{\cos(\frac{x}{2})}{\frac{x}{2} - \frac{\pi}{2}} \right)$

4.27) (2006-1) $\lim\limits_{x\to 2} \dfrac{\sqrt{x^2+1} - \sqrt{x}}{x^3 - 8}$

Limite e Continuidade

Seja $a \in D(f)$.
f ***é contínua em*** a***, se, e somente se, existe*** $\lim\limits_{x\to a} f(x)$ ***e*** $\lim\limits_{x\to a} f(x) = f(a)$

5. Verifique, justificando, se f é contínua em $x = x_0$.

5.1) (1999 – 2) $f(x) = \begin{cases} \dfrac{1-\cos x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$; $x_0 = 0$.

5.2) (1999 – 2) $f(x) = \begin{cases} \dfrac{\operatorname{sen} x}{x}, & x < 0 \\ \cos x, & x \geq 0 \text{ e } x \neq \pi \\ 1, & x = \pi \end{cases}$; $x_0 = 0$.

5.3) (2000 – 1) $f(x) = \begin{cases} \dfrac{\operatorname{sen}(x+3)}{x^2 - 9}, & x > -3 \\ 1/2, & x = -3 \\ \dfrac{\cos(\pi x)}{6}, & x < -3 \end{cases}$; $x_0 = -3$.

5.4) (2006 – 1) $f(x)=\begin{cases} \dfrac{x^3-x^2-10x-8}{x^2-2x-3}, \text{ se } x<3 \text{ e } x\neq -1 \\ 5/4, \text{ se } x=-1 \\ (x-6)\ln(x-3), \text{ se } x>3 \\ 2, \text{ se } x=3 \end{cases}$; $x_0=-1$ e $x_0=3$.

6. Determine, se possível, as constantes de modo que

6.1) (1999 – 2) $\lim\limits_{x\to 6}\dfrac{x^2+ax+b}{x-6}=8$.

6.2) (1999 – 1) $f(x)=\begin{cases} \dfrac{x^2+ax+b}{x+1}, & x<-1 \\ c, & x=-1 \\ x^2+5x, & x>-1 \end{cases}$ seja contínua em $x_o=-1$.

6.3) (2000 – 1) $f(x)=\begin{cases} 3x-2, & x>1 \\ ax, & x=1 \\ 5b-ax, & x<1 \end{cases}$ seja contínua em $x_o=1$.

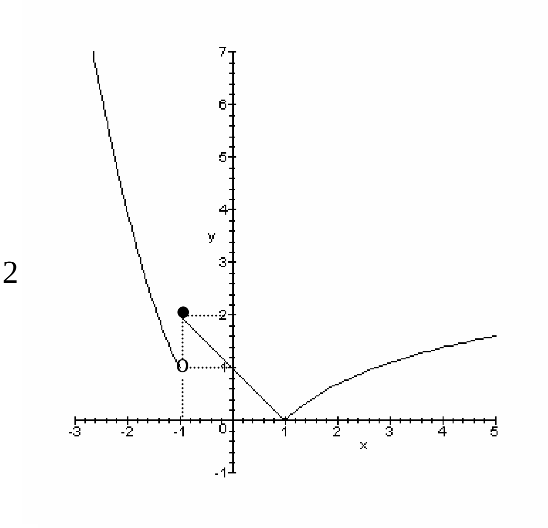
6.4) (2000 – 1) $f(x)=\begin{cases} mx^3-1, & x<1 \text{ e } x\neq 0 \\ x^2-m, & x\geq 1 \\ n-m, & x=0 \end{cases}$ seja contínua em $x_o=0$ e $x_1=1$.

6.5) (1999 – 2) $f(x)=\begin{cases} e^{2x}-1, & x<0 \\ 2x+a, & 0\leq x\leq 2 \\ \dfrac{x^2-b}{x-2}, & x>2 \end{cases}$ seja contínua em $x_o=0$ e $x_1=2$.

6.6) (2005 -2) Estude a continuidade da função $f(x)=\begin{cases} \dfrac{x^3-2x^2+x-2}{x^2-7x+10}, & x<3 \text{ e } x\neq 2 \\ -5/3, & x=2 \\ (x+1)\log_3|x-4|, & x\geq 3 \text{ e } x\neq 4 \end{cases}$.

RESPOSTAS

1. $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} (x^2) = +\infty$



$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} (\log x) = +\infty$

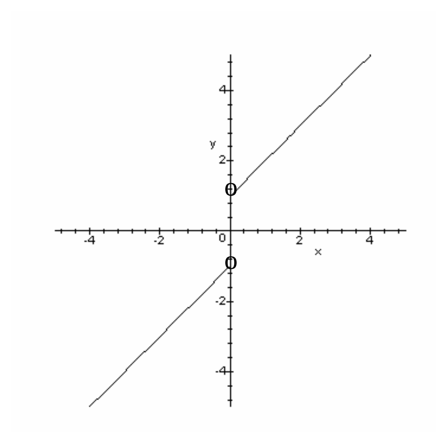
$\lim\limits_{x\to-1^-} f(x) = \lim\limits_{x\to-1^-} (x^2) = 1$, $\lim\limits_{x\to-1^+} f(x) = \lim\limits_{x\to-1^+} (-x+1) = 2$
logo $\nexists \lim\limits_{x\to-1} f(x)$

$\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} (-x+1) = 1$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (-x+1) = 0$, $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} (\log x) = 0$, logo $\lim\limits_{x\to 1} f(x) = 0$.

2.

Gráfico de $f(x)$:



$$f(x) = \begin{cases} x+1, & x > 0 \\ x-1, & x < 0 \end{cases}$$

$D(f) = IR^*$;

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (x-1) = -1$, $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} (x+1) = +1$,
logo $\nexists \lim\limits_{x\to 0} f(x)$.

3) a = 3, b = - 4 e c = 1

4.4.1) $\lim\limits_{x\to\frac{\pi}{2}^+} tgx = -\infty \Rightarrow \lim\limits_{x\to\frac{\pi}{2}^+} \left(\frac{1}{2}\right)^{tgx} = +\infty$ e $\lim\limits_{x\to\frac{\pi}{2}^-} tgx = +\infty \Rightarrow \lim\limits_{x\to\frac{\pi}{2}^-} \left(\frac{1}{2}\right)^{tgx} = 0$.

Logo $\nexists \lim\limits_{x\to\frac{\pi}{2}} \left(\frac{1}{2}\right)^{tgx}$.

4.2) –1.

4.3) $\lim\limits_{x\to 1^-} \dfrac{x^2-7x+6}{x^3-3x+2} = +\infty$ e $\lim\limits_{x\to 1^+} \dfrac{x^2-7x+6}{x^3-3x+2} = -\infty$, logo $\nexists \lim\limits_{x\to 1} \dfrac{x^2-7x+6}{x^3-3x+2}$.

4.4) $-\infty$. 4.5) 1/2 . 4.6) 1/6 . 4.7) $\sqrt{2}/6$. 4.8) 0 .

4.9) $\lim\limits_{x\to-\infty} (\sqrt{x^2+2}+x)=0$, logo $\lim\limits_{x\to-\infty} \ln(\sqrt{x^2+2}+x)=-\infty$.

4.10) 1 . 4.11) 0 . 4.12) 1/2 . 4.13) $-\infty$.

4.14) $\lim\limits_{x\to-1^-} \dfrac{2^x-1}{x^3-x}=+\infty$ e $\lim\limits_{x\to-1^+} \dfrac{2^x-1}{x^3-x}=-\infty$, logo $\nexists \lim\limits_{x\to-1} \dfrac{2^x-1}{x^3-x}$.

4.15) $\lim\limits_{x\to-1^-} \dfrac{\sqrt{3x^2+x}-2}{x^3+1}=+\infty$ e $\lim\limits_{x\to-1^+} \dfrac{\sqrt{3x^2+x}-2}{x^3+1}=-\infty$, logo $\nexists \lim\limits_{x\to-1} \dfrac{\sqrt{3x^2+x}-2}{\left|x^3+1\right|}$.

4.16) 1/2 4.17) 2 4.18) ½ 4.19) -1 4.20) 0

4.21) 0 4.22) 0 4.23) 0 4.24) 4 4.25) $\dfrac{4}{\pi^2}$

4.26) -1 4.27) $\lim\limits_{x\to2^-} \dfrac{\sqrt{x^2+1}-\sqrt{x}}{x^3-8}=-\infty$ e $\lim\limits_{x\to2^+} \dfrac{\sqrt{x^2+1}-\sqrt{x}}{x^3-8}=+\infty$, logo $\nexists \lim\limits_{x\to2} \dfrac{\sqrt{x^2+1}-\sqrt{x}}{x^3-8}$.

5.1) não 5.2) sim 5.3) não 5.4) $x_0 = -1$ sim, $x_0 = 3$ não

6.1) a = - 4, b = -12 6.2) a = -2, b = -3, c = -4 6.3) a = 1, b = 2/5

6.4) m = 1, n= 2 6.5) a = 0, b = 4

6.6) D(f) = R – {4}, f é contínua em seu domínio, exceto para x = 3.