UNIVERSIDADE FEDERAL DA BAHIA
INSTITUTO DE MATEMÁTICA
DEPARTAMENTO DE MATEMÁTICA
MAT A02 – CÁLCULO A

3ª LISTA DE EXERCÍCIOS - (QUESTÕES DE PROVA)

Integral Indefinida

1) Resolva as seguintes integrais indefinidas:

1.1) (1998 – 1) $\int \frac{\operatorname{sen}(1+3\ln x)}{x}\,dx$

1.2) (1998 – 1) $\int \frac{1+2e^{2x}}{\sqrt{x+e^{2x}}}\,dx$

1.3) (1998 – 1) $\int \frac{dx}{x(\ln x+3)}$

1.4) (1998 – 1) $\int \frac{1+\sqrt[3]{tg\,x}}{\cos^2 x}\,dx$

1.5) (1998 – 1) $\int \frac{\operatorname{sen} 2x}{\sqrt{1+\operatorname{sen}^2 x}}\,dx$

1.6) (1999 – 1) $\int \left(\frac{3x}{x^2+1}+x^2\,e^{x^3}\right)dx$

1.7) (1999 – 1) $\int (x^2\sqrt{x+1})\,dx$

1.8) (1999 – 1) $\int \left[\frac{2}{5}\left(\frac{1}{1+16x^2}+\frac{\cos x}{\operatorname{sen}^3 x}\right)\right]dx$

1.9) (1999 – 1) $\int (x\sqrt[3]{1-3x})\,dx$

1.10) (1999 – 2) $\int \frac{\operatorname{sen}^5(\sqrt[3]{x})\,\cos(\sqrt[3]{x})}{3\sqrt[3]{x^2}}\,dx$

1.11) (1999 – 2) $\int \frac{1+x}{1+x^2}\,dx$

1.12) (1999 –2) $\int \left(\frac{e^x}{e^{2x}+16}+\frac{1}{\sqrt{x}}\right)dx$

1.13) (1999 – 2) $\int [3^x+(3^x)^2+(3^x)^3]\,dx$

1.14) (1999 – 2) $\int \frac{dx}{\sqrt[3]{tg\,x}\cos^2 x}$

1.15) (2006-2) $\int ((x-4)sen(2x))dx$

1.16) (2006-2) $\int \frac{\left((\ln(x))^2+8\right)}{x\left((\ln(x))^3+8\right)}dx$

1.17) (2006-2) $\int \frac{(x-3)^2\sqrt{-x^2+6x}}{9-(x-3)^2}\,dx$

1.18) (2006-2) $\int (2x+1)\cos(2x)dx$

1.19) (2006-2)
$\int \frac{\left(3(\ln(x))^2+3\ln(x)-1\right)}{x(\ln(x)-1)\left((\ln(x))^2+2\ln(x)+2\right)}dx$

1.20) (2008-1) $\int x^3 sen(x^2)dx$

1.21) (2008-1) $\int \frac{dx}{(x+2)^4\sqrt{x^2+4x+8}}$ 1.22) (2008-2) $\int \frac{x^3+6x^2+3x+16}{x^3+4x}dx$

2.1) (1999 – 2) Calcule $f(x)$, sabendo que $f(\frac{\pi}{4})=3$ e que $\int \cos^2 x\, f'(x)\,dx = x+C$, sendo C uma constante real.

2.2) (1998 – 2) Determine a função f sabendo que $f(0) = 3$ e sua derivada f' é contínua e satisfaz a equação $\int e^{x^2} f'(x)\,dx = x^2 + C$.

2.3) (1998 – 2) Determine a função f sabendo que $f(0) = 3/4$ e sua derivada f' é contínua e satisfaz a equação $\int \text{arcsen}\,(x + f'(x))\,dx = 2x^2 + C$.

2.4) (2006 -2) Determine a função f(x) tal que $\int \frac{f'(x)}{\cot g(x)}dx = \ln(sen(x)) + C$, $f\left(\frac{\pi}{4}\right) = -\frac{\pi}{4}$.

3) Calcule a área da região do plano limitada pelas curvas:

3.1) (2006-2) $x = -y^2$, $x - y = 4$, $y = -1$ e $y = -2$

3.2) (2006-2) $x + 1 = (y-1)^2$ e $x = 2y - 3$

3.3) (2008-1) $x = -3y^2 + 4$ e $x = y^3$

3.4) (2008-2) $y = \sqrt{x}$, $x + y = 6$ e $x = 1$.

4) Calcule o valor médio de f(x) no intervalo indicado:

4.1) (2006-2) f(x) = arctg(2x) no intervalo $\left[0, \frac{1}{2}\right]$.

4.2) (2008 -1) $f(x) = \frac{1}{4+5\cos x}$ no intervalo $\left[0, \frac{\pi}{2}\right]$.

4.3) (2008-2) $f(x) = (\ln x)^2$ no intervalo [1, e].

5)(2008-1) Calcule $\lim_{x\to 0^+} \frac{\int_0^{x^2} sen\left(\sqrt{t}\right)dt}{x^3}$

ANEXO

TABELA DE INTEGRAIS IMEDIATAS ** Nesta tabela u é função derivável de x e C, α e a são constantes reais, com $a>0$ e $\alpha \neq -1$ **	
(1) $\int \mathrm{du} = \mathrm{u} + \mathrm{C}$	(2) $\int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1} + C$,
(3) $\int \frac{du}{u} = \ln \lvert u \rvert + C$	(4) $\int e^{u} du = e^{u} + C$
(5) $\int a^{u} du = \frac{a^{u}}{\ln a} + C$	(6) $\int \mathrm{sen\,u\,du} = -\cos \mathrm{u} + \mathrm{C}$
(7) $\int \cos u \, du = \mathrm{sen}\, u + C$	(8) $\int \sec^2 u \; du = tg\, u + C$
(9) $\int \cos ec^2 u \, du = -\cot g\, u + C$	(10) $\int \sec u \, tg\, u \, du = \sec u + C$
(11) $\int \cos ec\, u \;\; \cot g\, u \, du = -\cos ec\, u + C$	(12) $\int tg\, u \, du = \ln \lvert \sec u \rvert + C$
(13) $\int \cot g\, u \, du = \ln \lvert \mathrm{sen}\, u \rvert + C$	(14) $\int \sec u \, du = \ln \lvert \sec u + tg\, u \rvert + C$
(15) $\int \cos \sec u \, du = \ln \lvert \cos \sec u - \cot g\, u \rvert + C$	(16) $\int \frac{du}{a^2 + u^2} = \frac{1}{a} arctg\left(\frac{u}{a}\right) + C$
(17) $\int \frac{du}{\sqrt{a^2 - u^2}} = arcsen\left(\frac{u}{a}\right) + C$	(18) $\int \frac{du}{u\sqrt{u^2 - 1}} = arc \sec \lvert \mathrm{u} \rvert + C$
(19) $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{u-a}{u+a} \right\rvert + C$	(20) $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln \left\lvert u + \sqrt{u^2 \pm a^2} \right\rvert + C$

RESPOSTAS

1)

1.1) $-\frac{1}{3}\cos(1+3\ln x) + C$

1.2) $2\sqrt{x + e^{2x}} + C$

1.3) $\ln \lvert \ln x + 3 \rvert + C$

1.4) $tg\, x + \frac{3}{4}\sqrt[3]{tg^4 x} + C$

1.5) $2\sqrt{1 + \mathrm{sen}^2 x} + C$

1.6) $\frac{3}{2}\ln(x^2+1) + \frac{1}{3}e^{x^3} + C$

1.7) $\frac{2}{7}\sqrt{(x+1)^7} - \frac{4}{5}\sqrt{(x+1)^5} + \frac{2}{3}\sqrt{(x+1)^3} + C$

1.8) $\frac{1}{10} arctg\,(4x) - \frac{1}{5\,\mathrm{sen}^2 x} + C$

1.9) $\frac{1}{21}\sqrt[3]{(1-3x)^7} - \frac{1}{12}\sqrt[3]{(1-3x)^4} + C$

1.10) $\frac{\text{sen}^6(\sqrt[3]{x})}{6} + C$

1.11) $\text{arctg x} + \frac{1}{2}\ln(1+\text{x}^2) + \text{C}$

1.12) $\frac{1}{4}arctg\left(\frac{e^x}{4}\right) + 2\sqrt{\text{x}} + C$

1.13) $\left(\frac{3^x}{\ln 3} + \frac{(3^x)^2}{2\ln 3} + \frac{(3^x)^3}{3\ln 3}\right) + C$

1.14) $\frac{3}{2}\sqrt[3]{tg^2 x} + C$

1.15) $-\frac{(x-4)}{2}\cos(2x) + \frac{sen(2x)}{4} + C$

1.16) $\ln|\ln x + 2| + \frac{2}{\sqrt{3}}arctg\left(\frac{\ln x - 1}{\sqrt{3}}\right) + C$

1.17) $\frac{9}{2}arcsen\left(\frac{x-3}{3}\right) - \frac{(x-3)}{2}\sqrt{-x^2+6x} + C$

1.18) $\frac{(2x+1)}{2}sen(2x) + \frac{\cos(2x)}{2} + C$

1.19)

$\ln|\ln x - 1| + \ln|(\ln x)^2 + 2\ln x + 2| + arctg(\ln x + 1) + C$

1.20) $-\frac{x^2\cos(x^2)}{2} + \frac{sen(x^2)}{2} + C$

1.21) $-\frac{\left(\sqrt{x^2+4x+8}\right)^3}{48(x+2)^3} + \frac{\sqrt{x^2+4x+8}}{16(x+2)} + C$

1.22) $x + 4\ln|x| + \ln|x^2+4| - \frac{1}{2}arctg\left(\frac{x}{2}\right) + C$

2) 2.1) $f(x) = tg\,x + 2$ 2.2) $f(x) = -e^{-x^2} + 4$ 2.3) $f(x) = -\frac{1}{4}\cos(4x) - \frac{x^2}{2} + 1.$

2.4) $f(x) = \cot g(x) - x + 1$

3) 3.1) $\frac{33}{2}$ u.a 3.2) $\frac{4}{3}$ u. a 3.3) $\frac{27}{4}$ u.a 3.4) $\frac{35}{6}$ u.a.

4) 4.1) $\frac{\pi}{4} - \frac{\ln 2}{2}$ 4.2) $\frac{2\ln(2)}{3\pi}$ 4.3) $\frac{e-2}{e-1}$

5) 2/3